

Macroscopic quantum oscillator based on a flux qubit

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In this paper a macroscopic quantum oscillator is introduced that consists of a flux qubit in the form of a cantilever. The magnetic flux linked to the flux qubit and the mechanical degrees of freedom of the cantilever are naturally coupled. The coupling is controlled through an external magnetic field. The ground state of the introduced flux-qubit-cantilever corresponds to a quantum entanglement between magnetic flux and the cantilever displacement.

I. INTRODUCTION

Realization of quantum superposition and quantum entanglement of macroscopically distinct quantum states or Schrödinger cat states is one of the main objectives of quantum experiments to explore quantum physics at a macroscopic scale [1]. Magnetic flux quantization where a net magnetic flux threading a closed superconducting loop is quantised signifies a quantum effect at a mesoscopic scale. From foundational point of view, a quantum superposition of distinct magnetic flux states is discussed in ref [1]. Experimentally, a quantum superposition of distinct magnetic flux states has been realised through a superconducting loop interrupted by Josephson junctions - a quantum device known as a flux qubit [2–4]. Flux qubits that are considered as designed macroscopic atoms have been studied theoretically and experimentally in context of implementation of a quantum computer [5–7]. On the other hand, a micro/nanomechanical cantilever [8] that is regarded as a macroscopic quantum harmonic oscillator exhibits a quantum dynamics at a low temperature. Such a cantilever when strongly coupled to another quantum system can produce a quantum entanglement at a macroscopic scale. Various different approaches are being explored through experiments to strongly couple a micro/nanomechanical cantilever to another quantum system such as photons in a cavity [9–15], a nitrogen vacancy [16], a Bose Einstein condensate [17, 18] and superconducting quantum circuits [19–22].

In this paper, a macroscopic quantum oscillator based on a flux qubit is introduced. The quantum oscillator that is named a flux-qubit-cantilever consists of a flux qubit where a part of the flux qubit loop is in the form of a cantilever. The magnetic flux threading the flux qubit and the mechanical degrees of freedom of the cantilever are naturally coupled to each other. The coupling is controlled through an external magnetic field. The potential energy profile of the flux-qubit-cantilever is tunable from a single potential well to a double well potential. The resulting ground state of the flux-qubit-cantilever is a quantum entangled state between magnetic flux and de-

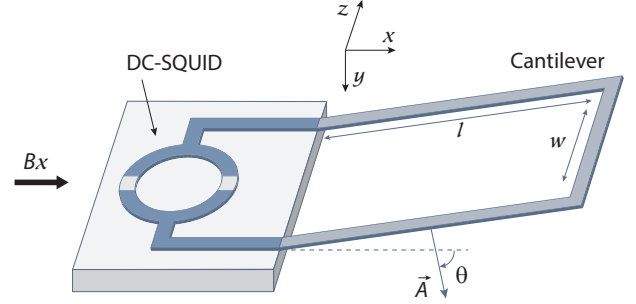


FIG. 1. A schematic of a flux-qubit-cantilever. A part of the flux qubit (larger loop) is in the form of a cantilever. External magnetic field B_x controls the coupling between the flux qubit and the cantilever. An additional magnetic flux threading a DC-SQUID (smaller loop) that consists of two Josephson junctions adjusts the tunneling amplitude. DC-SQUID can be shielded from the effect of B_x .

flection of the cantilever that correspond to macroscopic observables.

The phenomenon of flux quantization is a consequence of Aharonov-Bohm effect. The persistent current flowing through a superconducting loop is proportional to $\nabla\varphi(r) - (2e/\hbar)A(r)$, where $\varphi(r)$ is the phase of the Cooper pair wavefunction, $A(r)$ is the vector potential, e is charge on electron and \hbar is Plank 's constant ($\hbar = h/2\pi$). Due to Meissner effect the persistent current decreases exponentially from surface of a superconductor. Therefore, $\nabla\varphi(r) = (2e/\hbar)A(r)$ for any point r situated within the loop at a distance from the surface much greater than the penetration depth. To maintain a single valued Cooper pair wavefunction the total phase accumulated over a closed path is an integral multiple of 2π that results a quantization of net magnetic flux linked to a superconducting loop [23].

For a superconducting loop interrupted by a Josephson junction a phase $\Delta\varphi$ is accumulated across the Josephson junction. Therefore, the net phase accumulated around a closed path is $\Delta\varphi + 2\pi\Phi/\Phi_0$. To maintain continuity of the Cooper pair wavefunction the phase accumulated

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around a closed path passing through the superconducting loop and the Josephson junction is an integral multiple of 2π *i.e.* $\Delta\varphi + 2\pi\Phi/\Phi_o = 2k\pi$, where a variable Φ is the magnetic flux that threads the flux qubit loop. The potential energy of a superconducting loop interrupted by a single Josephson junction (flux qubit) consists of two components, the first component is magnetic energy stored in the superconducting loop due to the magnetic flux Φ threading through it in the presence of an external magnetic flux Φ_a and the second component is the potential energy of the Cooper pairs while tunneling through the Josephson junction. The Josephson junction also forms a junction capacitor and for a flux qubit the energy contribution due to energy stored at Josephson junction capacitor is considered to be much less than the inductive energy. Therefore, the Hamiltonian of a flux qubit is [6, 23]

$$H_Q = \frac{p_\Phi^2}{2C} + \frac{(\Phi - \Phi_a)^2}{2L} + E_j (1 - \cos(2\pi\Phi/\Phi_o)) \quad (1)$$

where $p_\Phi = i\hbar\partial/\partial\Phi$ is the momentum conjugate to Φ , the second term $(\Phi - \Phi_a)^2/2L$ is the magnetic energy stored in the flux qubit loop of self inductance L for an external flux Φ_a , the third term $E_j(1 - \cos(2\pi\Phi/\Phi_o))$ is the Josephson energy term with Josephson energy $E_j = I_c\hbar/2e$, $\Phi_o = h/2e$ is the flux quantum and I_c is the critical current *i.e.* the maximum current that can pass through a Josephson junction without dissipation. The potential energy of a flux qubit corresponds to a symmetric double well potential if qubit is biased at half of a flux quantum such that $\Phi_a = \Phi_o/2$.

Consider a schematic of a flux-qubit-cantilever shown in Fig. 1 where a part of a superconducting loop of a flux qubit forms a cantilever. The larger loop is interrupted by a smaller loop consisting of two Josephson junctions - a DC Superconducting Quantum Interference Device (DC-SQUID). The Josephson energy that is constant for a single Josephson junction can be varied by applying a magnetic flux to a DC-SQUID loop. However, for calculations a flux qubit with a single Josephson junction is considered throughout this paper. The external magnetic flux applied to the cantilever is $\Phi_a = B_x A \cos(\theta)$, where B_x is the magnitude of an uniform external magnetic field along x -axis and area vector \vec{A} subtends an angle θ with the magnetic field direction (x -axis). Consider the cantilever oscillates about an equilibrium angle θ_0 with an intrinsic frequency of oscillation ω_i *i.e.* the frequency in absence of magnetic field. The external magnetic flux applied to the flux-qubit-cantilever depends on the cantilever deflection therefore, the flux qubit whose potential energy depends on an external flux is coupled to the cantilever degrees of freedom. The potential energy of the flux-qubit-cantilever corresponds to a two dimensional potential $V(\Phi, \theta)$ and the Hamiltonian of the flux-qubit-cantilever interrupted by a single Josephson

junction is

$$H = \frac{p_\theta^2}{2I_m} + \frac{1}{2}I_m\omega_i^2(\theta - \theta_0)^2 + \frac{p_\Phi^2}{2C} + \frac{(\Phi - B_x A \cos(\theta))^2}{2L} + E_j (1 - \cos(2\pi\Phi/\Phi_o)) \quad (2)$$

The first two terms of Eq. 2 correspond to a Hamiltonian of the cantilever, last three terms correspond to a Hamiltonian of a flux qubit and its coupling to the cantilever. Fourth term is the magnetic energy of a superconducting loop in presence of an external field, fifth term is the Josephson energy term of a Josephson junction. Where Φ is the magnetic flux threading the superconducting loop of flux qubit, $p_\theta = i\hbar\partial/\partial\theta$ is the momentum conjugate to θ , I_m is the moment of inertia of the cantilever about z -axis, $A(l \times w)$ is area of the cantilever, l is length and w is width of the cantilever. In case of $B_x = 0$, the coupling between the mechanical degrees of freedom of the cantilever and flux states of the flux qubit is zero. For $\omega_i = 0$ the cantilever has zero intrinsic restoring torque about z -axis, however due to magnetic and Josephson energy there exist several potential energy minima forming a two dimensional multi-well potential.

The two dimensional potential energy of a flux-qubit-cantilever, $V(\Phi, \theta) = (\Phi - B_x A \cos(\theta))^2/2L + E_j (1 - \cos(2\pi\Phi/\Phi_o)) + I_m\omega_i^2(\theta - \theta_0)^2/2$ has a single global minimum situated at $(n\Phi_o, \theta_n^+)$ if the equilibrium position of the cantilever is $\theta_0 = \theta_n^+$, where $\theta_n^+ = +\cos^{-1}[n\Phi_o/B_x A]$, $m\Phi_o < B_x A < (m+1)\Phi_o$, integer $m > 0$ and $n = -m, -m+1, \dots, m-1, m$. The potential energy near the global minimum point is regarded as a single two dimensional potential well. Similarly, a global minimum of $V(\Phi, \theta)$ is situated at $(n\Phi_o, \theta_n^-)$ if the equilibrium position of the cantilever is chosen $\theta_0 = \theta_n^-$, where $\theta_n^- = -\cos^{-1}[n\Phi_o/B_x A]$. The location of a potential energy minimum depends on the cantilever equilibrium angle and the later can be fine tuned by tilting the magnetic field *w.r.t* x -axis. Therefore, from Taylor expansion of $V(\Phi, \theta)$ around a potential minimum $(n\Phi_o, \theta_n^+)$ of a single well the Hamiltonian Eq. 2 is

$$H_n \simeq \frac{p_\phi^2}{2C} + \frac{p_\delta^2}{2I_m} + \left(\frac{1}{2L} + \frac{2\pi^2 E_j}{\Phi_o^2} \right) \phi^2 + \left(\frac{B_x^2 A^2 - n^2 \Phi_o^2}{2L} + \frac{1}{2} I_m \omega_i^2 \right) \delta^2 + \frac{(B_x^2 A^2 - n^2 \Phi_o^2)^{1/2}}{L} \phi \delta \quad (3)$$

Where angle $\delta = \theta - \theta_n^+$ and flux $\phi = \Phi - n\Phi_o$ are defined *w.r.t* the potential well minimum $(n\Phi_o, \theta_n^+)$. The momentum conjugate to ϕ and δ are $p_\phi = i\hbar\partial/\partial\phi$ and $p_\delta = i\hbar\partial/\partial\delta$, respectively. The Hamiltonian Eq. 3 corresponds to a two coupled quantum harmonic oscillators of non identical masses and different spring constants where the last term containing a product of ϕ and δ is the coupling between oscillators. Due to the term containing a product of ϕ and δ the eigen states of the Hamiltonian Eq. 3 cannot be written as a product of the magnetic

flux states and the cantilever oscillator states *i.e.* eigen functions of Eq. 3 are non-separable functions of ϕ and δ . On the other hand, if the cantilever equilibrium angle is chosen $\theta_0 = \theta_n^-$, such that the global potential minimum is located at $(n\Phi_o, \theta_n^-)$, the sign of the product term of Eq. 3 is reversed.

The Hamiltonian Eq. 3 is written as

$$H_n \simeq \frac{p_\phi^2}{2C} + \frac{p_\delta^2}{2I_m} + \frac{1}{2}C\omega_\phi^2\phi^2 + \frac{1}{2}I_m\omega_\delta^2\delta^2 + \kappa\phi\delta \quad (4)$$

Where oscillation frequencies along ϕ and δ are

$$\omega_\phi^2 = \frac{1}{C} \left(\frac{1}{L} + \frac{4\pi^2 E_j}{\Phi_o^2} \right) \quad \omega_\delta^2 = \left(\frac{B_x^2 A^2 - n^2 \Phi_o^2}{I_m L} + \omega_i^2 \right) \quad (5)$$

and the coupling constant κ (for $B_x A > n\Phi_o$) is

$$\kappa = \frac{(B_x^2 A^2 - n^2 \Phi_o^2)^{1/2}}{L} \quad (6)$$

The coupling constant κ increases with an external magnetic field B_x . It is important to notice that even if the intrinsic frequency ω_i of the cantilever is zero the cantilever experiences a restoring force resulting a nonzero ω_δ in presence of an external magnetic field.

Through a basis transformation

$$X = \left(\frac{C}{I_m} \right)^{1/4} \cos(\beta)\phi + \left(\frac{I_m}{C} \right)^{1/4} \sin(\beta)\delta \quad (7)$$

$$Y = - \left(\frac{C}{I_m} \right)^{1/4} \sin(\beta)\phi + \left(\frac{I_m}{C} \right)^{1/4} \cos(\beta)\delta \quad (8)$$

The Hamiltonian Eq. 4 corresponds to a Hamiltonian of two uncoupled oscillators of identical masses $\mu = (CI_m)^{1/2}$, such that

$$H_n = \frac{P_X^2}{2\mu} + \frac{P_Y^2}{2\mu} + \frac{1}{2}\mu\omega_X^2 X^2 + \frac{1}{2}\mu\omega_Y^2 Y^2 \quad (9)$$

for an angle of rotation of basis

$$\beta = \frac{1}{2} \tan^{-1} \left[\frac{2\kappa/\mu}{\omega_\phi^2 - \omega_\delta^2} \right] \quad (10)$$

Where $P_X = i\hbar\partial/\partial X$ and $P_Y = i\hbar\partial/\partial Y$ are the momentum conjugate to X and Y , respectively. The eigen frequencies of the uncoupled Hamiltonian Eq. 9 are

$$\omega_X^2 = \omega_\phi^2 \cos^2(\beta) + \omega_\delta^2 \sin^2(\beta) + \frac{\kappa}{\mu} \sin(2\beta) \quad (11)$$

and

$$\omega_Y^2 = \omega_\phi^2 \sin^2(\beta) + \omega_\delta^2 \cos^2(\beta) - \frac{\kappa}{\mu} \sin(2\beta) \quad (12)$$

The ground state wavefunction of uncoupled Hamiltonian Eq. 9 in X - Y basis is

$$\Psi_0^n(X, Y) = \left(\frac{\mu^2 \omega_X \omega_Y}{\pi^2 \hbar^2} \right)^{1/4} \exp \left(-\frac{\mu \omega_X X^2}{2\hbar} \right) \times \exp \left(-\frac{\mu \omega_Y Y^2}{2\hbar} \right) \quad (13)$$

The ground state wavefunction in ϕ - δ basis is

$$\begin{aligned} \Psi_0^n(\phi, \delta) = & \left(\frac{CI_m \omega_X \omega_Y}{\pi^2 \hbar^2} \right)^{1/4} \times \\ & \exp \left(-\frac{C}{2\hbar} (\omega_X \cos^2(\beta) + \omega_Y \sin^2(\beta)) \phi^2 \right) \\ & \times \exp \left(-\frac{I_m}{2\hbar} (\omega_X \sin^2(\beta) + \omega_Y \cos^2(\beta)) \delta^2 \right) \\ & \times \exp \left(-\frac{(CI_m)^{1/2}}{2\hbar} (\omega_X - \omega_Y) \sin(2\beta) \phi \delta \right) \end{aligned} \quad (14)$$

The last exponent of Eq. 14 consists of a product of ϕ and δ therefore, the ground state wavefunction $\Psi_0^n(\phi, \delta)$ is non separable *i.e.* $\Psi_0^n(\phi, \delta)$ cannot be written as a product of variable separated functions $\psi_0^n(\phi)$ and $\psi_0^n(\delta)$. The ground state wavefunction $\Psi_0^n(\phi, \delta)$ is separable if the coupling constant $\kappa = 0$.

The ground state $|\alpha\rangle_0^n$ of the flux-qubit-cantilever in the basis $|\phi\rangle|\delta\rangle$ is

$$|\alpha\rangle_0^n = \int \int C_{\phi,\delta}^n |\phi\rangle|\delta\rangle d\phi d\delta \quad (15)$$

Where $C_{\phi,\delta}^n = (\langle \delta | \langle \phi |) |\alpha\rangle_0^n = \Psi_0^n(\phi, \delta)$. Since the ground state wave-function $\Psi_0^n(\phi, \delta)$ of the flux-qubit-cantilever is non-separable therefore, the ground state $|\alpha\rangle_0^n$ is an entangled state.

To estimate the oscillation frequencies, consider a flux-qubit-cantilever made of niobium which is a type-II superconductor of transition temperature 9.26 K. Consider niobium has a square cross-section with thickness $t = 0.5\mu\text{m}$, $l = 6\mu\text{m}$, $w = 4\mu\text{m}$ ($A = l \times w$). For these dimensions the mass of the cantilever is $3.64 \times 10^{-14}\text{Kg}$ and moment of inertia is $I_m \simeq 7.28 \times 10^{-25}\text{Kgm}^2$. The critical current of Josephson junction $I_c = 5\mu\text{A}$, capacitance $C = 0.1\text{pF}$ and self inductance $L = 100\text{pH}$ are of the same order as described in Ref [2]. The quantity $\beta_L = 2\pi LI_c/\Phi_o \simeq 1.52$. Consider intrinsic frequency of the cantilever is $\omega_i = 2\pi \times 12000\text{ rad/s}$. For an equilibrium angle $\theta_0 = \theta_n^+ = \cos^{-1}[n\Phi_o/B_x A]$ there exists a single global potential energy minimum. If we consider $n = 0$ and $B_x = 5 \times 10^{-2}\text{T}$ the global potential energy minimum is located at $(0, \pi/2)$. For parameters described above $\omega_\phi \simeq 2\pi \times 7.99 \times 10^{10}\text{ rad/s}$, $\omega_\delta = 2\pi \times 25398.1\text{ rad/s}$, $\kappa = 0.012\text{ A}$. The eigen frequencies of the flux-qubit-cantilever are $\omega_X \simeq 2\pi \times 7.99 \times 10^{10}\text{ rad/s}$ and $\omega_Y = 2\pi \times 21122.5\text{ rad/s}$. A contour plot of potential energy of the flux-qubit-cantilever, indicating a two dimensional global minimum located at $(0, \pi/2)$ and two local minima, is shown in Fig. 2. Even if we consider intrinsic frequencies to be zero the restoring force is still nonzero due to a finite coupling constant. For $\omega_i = 0$, the angular frequencies are $\omega_\phi \simeq 2\pi \times 7.99 \times 10^{10}\text{ rad/s}$, $\omega_\delta = 2\pi \times 22384.5\text{ rad/s}$, $\omega_X \simeq 2\pi \times 7.99 \times 10^{10}\text{ rad/s}$ and $\omega_Y = 2\pi \times 17382.8\text{ rad/s}$. The frequencies can be increased by increasing external magnetic field and by decreasing the dimensions of the cantilever that reduces mass and moment of inertia.

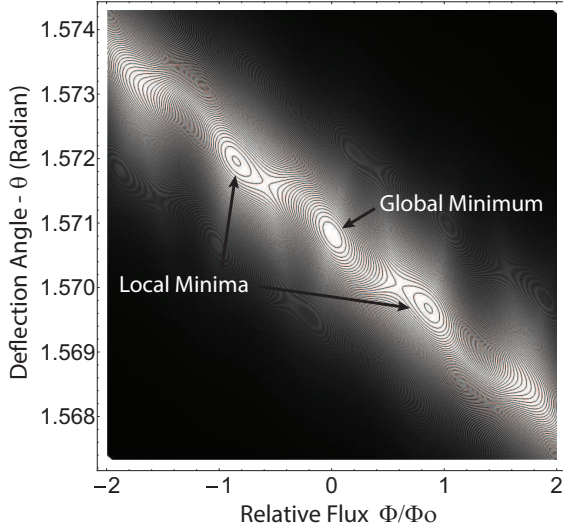


FIG. 2. A contour plot indicating location of a two dimensional global potential energy minimum at $(0, \pi/2)$ and local minima for cantilever equilibrium angle $\theta_0 = \pi/2$, $\omega_i = 2\pi \times 12000$ rad/s, $B_x = 5 \times 10^{-2}$ T. The contour interval in units of frequency (E/h) is $\sim 3.9 \times 10^{11}$ Hz.

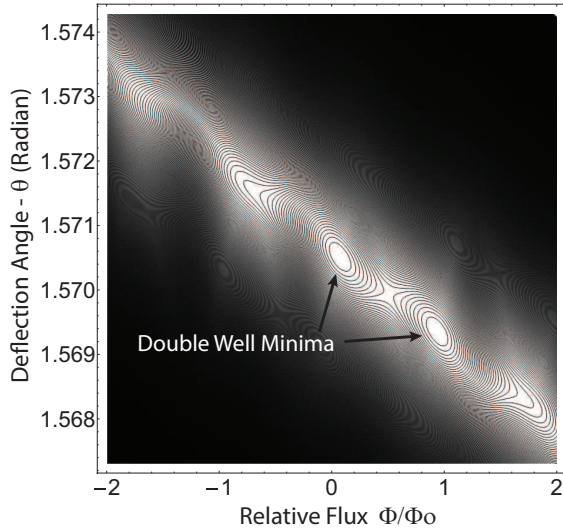


FIG. 3. A contour plot indicating location of two dimensional potential energy minima forming a symmetric double well potential for cantilever equilibrium angle $\theta_0 = \cos^{-1}[\Phi_0/2B_xA]$, $\omega_i = 2\pi \times 12000$ rad/s, $B_x = 5 \times 10^{-2}$ T. The contour interval in units of frequency (E/h) is $\sim 4 \times 10^{11}$ Hz.

The potential energy of the flux-qubit-cantilever corresponds to a symmetric double well potential (*i.e.* two global minima and $m\Phi_0 < B_xA < (m+1)\Phi_0$) if equilibrium angle of the cantilever $\theta_0 = \cos^{-1}[(2n+1)\Phi_0/2B_xA]$, assuming ω_i is less than or of the order of the first term of ω_δ (Eq. 5) *i.e.* $((B_x^2A^2 - n^2\Phi_0^2)/I_mL)^{1/2}$. For a double well potential the left potential well is located near $(n\Phi_0, \theta_n^+)$ and the right potential well is located near $((n+1)\Phi_0, \theta_{n+1}^+)$. Any variation of the equilibrium angle around $\cos^{-1}[(2n+1)\Phi_0/2B_xA]$ introduces an asymmetry in the double well potential. Consider $n=0$ such that $\theta_0 = \cos^{-1}[\Phi_0/2B_xA]$ therefore, at equilibrium the flux-qubit-cantilever is biased at a half of a flux quantum, $\Phi_0/2$. A contour plot indicating a two dimensional symmetric double well potential is shown in Fig. 3. Consider the nonseparable ground states of the left and the right well are $|\alpha\rangle_L$ and $|\alpha\rangle_R$, respectively. The barrier height between the wells of the double well potential which is less than $2E_j$ reduces when ω_i is increased. The barrier height controls the tunneling between potential wells and it can also be tuned through an external magnetic flux applied to a DC-SQUID of the flux-qubit-cantilever. When tunneling between wells is introduced the ground state of flux-qubit-cantilever is $|\Psi\rangle_E = [|\alpha\rangle_L + |\alpha\rangle_R]/\sqrt{2}$. The state $|\Psi\rangle_E$ is an entangled state of distinct magnetic flux and distinct cantilever deflection states. The state $|\Psi\rangle_E$ can be realised by cooling the flux-qubit-cantilever to its ground state.

A special case if intrinsic frequency ω_i of the flux-qubit-cantilever is zero, the potential energy $V(\Phi, \theta) = (\Phi - B_xA \cos(\theta))^2/2L + E_j(1 - \cos(2\pi\Phi/\Phi_0))$ has multiple two-dimensional global minima forming a lattice. For $m\Phi_0 < B_xA < (m+1)\Phi_0$, integer $m > 0$, the minima of potential energy are located at $(n\Phi_0, \theta_n^+)$ and $(n\Phi_0, \theta_n^-)$, where $n = -m, -m+1, \dots, m-1, m$.

In this paper a macroscopic quantum oscillator (flux-qubit-cantilever) is introduced that exhibits a natural coupling between magnetic flux and cantilever deflection. The coupling constant can be varied through an external magnetic field. The potential energy is adjusted through the cantilever equilibrium position and an external magnetic field. The ground state of the flux-qubit-cantilever corresponds to a quantum entangled state between magnetic flux and cantilever deflection. The macroscopic quantum oscillator presented in this paper is worth exploring through experiments.

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